11.1 SEQUENCES

IF $\lim\limits_{n\to\infty} a_n$ EXISTS, WE SAY $\{a_n\}$ CONVERGES
OR IS CONVERGENT

DNE — DIVERGES
OR IS DIVERGENT

INCLUDES $=\infty$ OR $=-\infty$

SQUEEZE TH'M FOR SEQUENCES

IF $a_n \le b_n \le c_n$ FOR ALL $n \ge n_0$

AND $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = L$

THEN $\lim\limits_{n\to\infty} b_n = L$

CONSIDER $\left\{ \dfrac{2+\sin n}{n^2} \right\}$

$$-1 \le \sin n \le 1$$

$$2-1 \le 2+\sin n \le 2+1$$

$$\frac{1}{n^2} \le \frac{2+\sin n}{n^2} \le \frac{3}{n^2}$$

$\lim\limits_{n\to\infty} \dfrac{1}{n^2} = \lim\limits_{n\to\infty} \dfrac{3}{n^2} = 0$ SO BY SQUEEZE THEOREM

$$\lim_{n\to\infty} \frac{2+\sin n}{n^2} = 0$$

$\lim_{n\to\infty} r^n$ ~~DIVERGES~~ DNE IF $r>1$

$(r\in\mathbb{R})$

$= 0$ IF $0\le r<1$

$= 1$ IF $r=1$

DNE IF $r<-1$

$=0$ IF $-1<r<0$

DNE IF $r=-1$

$r>1$

$0<r<1$

$r=1$

$r^x = 1^x = 1$

$r=0$

$r^x = 0^x = 0$ IF $x>0$

IS UNDEFINED IF $x\le 0$

0^0 IS ALSO UNDEFINED

eg. $r=-3$

$\{-3, (-3)^2, (-3)^3, \ldots\}$

$\{-3, 9, -27, \ldots\}$

WHEN $r<0$

$$\left.\begin{array}{l} r^{\frac{3}{2}} = \sqrt{r^3} \\ r^{\frac{7}{4}} = \sqrt[4]{r^7} \end{array}\right\} \text{DNE IN } \mathbb{R}$$

eg. $r=-\frac{1}{2}$

$\{-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots\}$

$$\lim_{n\to\infty}\left|\left(-\tfrac{1}{2}\right)^n\right| = \lim_{n\to\infty}\left(\tfrac{1}{2}\right)^n = 0$$

so $\lim_{n\to\infty}\left(-\frac{1}{2}\right)^n = 0$

IF $r=-1$

$\{-1, 1, -1, 1, -1, 1, \ldots\}$

$\lim_{n\to\infty}(-1)^n$ DNE

NOTE: $\lim_{n\to\infty}|(-1)^n| = \lim_{n\to\infty} 1^n = 1 \neq 0$

CAN'T USE ABSOLUTE VALUE THEOREM

$$\lim_{n\to\infty} r^n = 0 \quad \text{IF } -1 < r < 1 \quad \text{IE, IF } |r| < 1$$

$$= 1 \quad \text{IF } r = 1$$

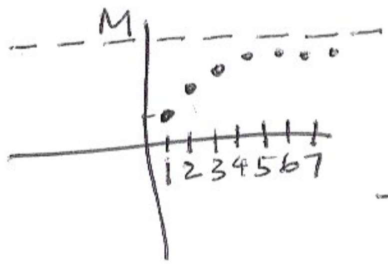
$$\text{DNE} \quad \text{IF } r \le -1 \text{ OR } r > 1 \quad \text{IE. } |r| > 1 \text{ OR } r = -1$$

$\{r^n\}$ CONVERGES IF $-1 < r \le 1$

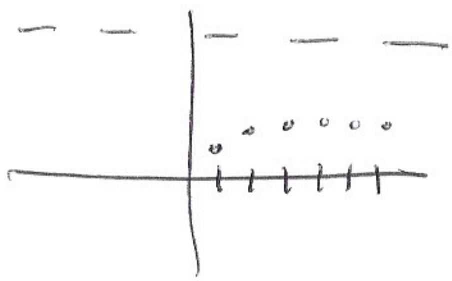
DIVERGES IF $r \le -1$ OR $r > 1$

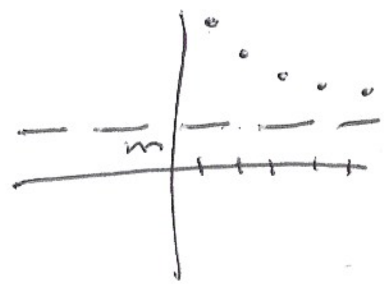
BELOW DECREASING

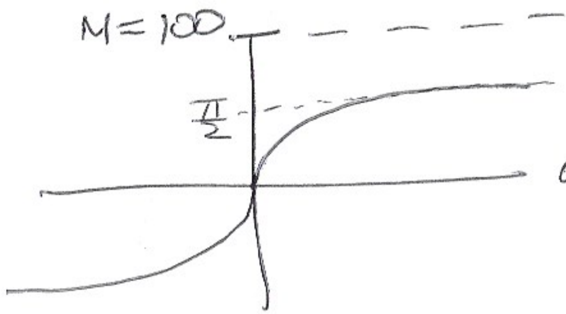
TH'M: IF A SEQUENCE IS BOUNDED ABOVE AND INCREASING THEN IT IS CONVERGENT



← SEQUENCE DOES NOT NECESSARILY CONVERGES TO M







arctan x

11.2 SERIES

A SERIES IS THE SUM OF THE TERMS OF A SEQUENCE

$$\sin\frac{1}{2} \approx \underset{n=1}{\frac{1}{2}} - \underset{n=2}{\frac{\left(\frac{1}{2}\right)^3}{3!}} + \underset{n=3}{\frac{\left(\frac{1}{2}\right)^5}{5!}} - \underset{n=4}{\frac{\left(\frac{1}{2}\right)^7}{7!}} + \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\left(\frac{1}{2}\right)^{2n-1}}{(2n-1)!}$$

$$\sin\frac{1}{2} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)!\,2^{2n-1}}$$

$$2^{2n-1}(2n-1)!$$

AMBIGUOUS